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MATHEMATICAL OPTIMIZATION METHODS APPLIED TO SHIP DESIGN

by

REUVEN LEOPOLD

October, 1965

Prepared under M.I.T. DSR No. 9971

Sponsored by the Bureau of Ships

Contract No. NObs 90100

Cambridge, Massachusetts

MASSACHUSETTS INSTITUTE OF TECHNOLOGY

DEPARTMENT OF NAVAL ARCHITECTURE AND MARINE ENGINEERING

July 4th 1997

To my friend Mike Bosworth

Reuven

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ABSTRACT

Various existing mathematical methods of functional optimization and optimization criteria were examined for application to the ship design problem. Based on this study, the Exponential Random Search technique in conjunction with "Multiple Parameter Weighting Criteria" were used in a sample design study of a cargo ship. Results of this study show that this approach to the ship design problem:

(1) yields better results than any other method available at the present time;

(2) arrives at a solution in less time than current methods;

(3) is more versatile than any other method available;

(4) performs the optimization more correctly because it does not neglect any of the features of the problem of optimizing a function of several variables.

The demonstrated flexibility, versatility, and efficiency of the chosen method constitute in the author's opinion, a powerful tool in the preliminary design of ships.

ACKNOWLEDGMENTS

I wish to thank Professor Philip Mandel for his continuous advice and thorough editorial help.

I also wish to thank Professor Ernst G. Frankel for his constructive ideas at the outset of this study.

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INTRODUCTION

The present state of the art in the ship design optimization problem is still of the "brute force" type. Some basic parameters affecting the size of the ship are chosen to be varied over a range in finite step sizes. Naturally, both the parameters and the variation range of their values are based on experience. The result is a multidimensional matrix that grows very fast, since it is a function of the number of variables, the step size, and the range chosen.

The specifications of ship requirements such as: speed, payload, endurance, etc., generally result in upper and lower limits on the various ship hull parameters such as length, beam, depth C_p, $C_{\bowtie}$, etc. The designer has to select the best combination of these parameters to satisfy a set criterion such as minimum initial cost or minimum overall cost including operational costs.

Therefore, the designer is faced now with the problem of evaluating the cost criterion as a function of the parameters

$$F = f\,(L,B,H,D,C_p,C_{\bowtie}\ \ldots.) \qquad (1)$$

subject to constraints resulting from various interrelations among the variables such as:

$$\begin{aligned} \varphi_1\ (L,B,H) &= 0 \\ \varphi_2\ (B,C_p\ \ldots.) &= 0 \end{aligned} \qquad (2)$$

and also subject to some boundary conditions resulting from rules, regulation, and past experience. Previous attempts (1) to find the optimum combination of these parameters were cumbersom and non elegant. For several reasons, they may not necessarily yield an optimum at all. First, the method might solve for a relative minimum and not for an absolute one, since the selection of the search area might exclude the absolute extremum. Second, the probability density function of the search is uniform, i.e. it acts like a pure random search with a rather low search efficiency.

For these reasons, the present effort attacks the problem first of all as a general optimization of a function, (Eq. 1), subject to constraints and boundary conditions. Several fields of science, such as automatic controls, operations research, design of experiments, to name a few, have come up with various methods coupled to some criteria, for functional optimization. This investigation examines the various mathematical techniques developed in some of these branches of engineering and science and applies the best, or the best combination of a few, to optimize the ship design. In order to optimize a design, we have to decide on the following:

1. Choice of the optimization technique.
2. Choice of an optimization criterion.
3. The mathematical model of the design process.

In the first category, there are numerous methods available that will be examined and compared for their relative merit in general and in particular concerning the ship design problem. To name a few:

1. Closed form maximum and minimum with Lagrange multipliers
2. Steepest ascent methods
3. Various random search techniques
4. Dynamic programming.

In the second category, the following criteria will be examined:

a. The sum RMS error of all independent variables
b. Multiple parameter weighting criteria
c. Max-Ranking criteria.

These are mathematical criteria in use in other fields that have to be examined to see if they have meaning in terms of the ship design problem.

The third category pertains to the various relationships particular to ships involving consideration in the fields of ship resistance, powering, structural weight, machinery weight, rules, regulations, etc.

There are basic differences between the ship design optimization problem and the other fields where these techniques have been applied. The differences are significant enough to cast doubt on whether some of the techniques are applicable at all. For example, let us compare the design optimization problem to an automatic control optimization problem:

1. The objective of a control optimization is to minimize the motion of the vehicle to which the control surface is attached.

The vehicle is subject to continuous disturbances as inputs. On the other hand, the design process is not subject to a continuous change in inputs once a choice of parameters is made.

2. The control problem is a time-varying process, while ship design is a stationary process.

3. The control problem must be approached from a statistical point of view, while the design problem requires deterministic results.

4. Perhaps the major difference is the fact that in the control optimization, the standard of performance is assumed as fixed in the beginning. This standard of performance may be maximum allowable acceleration, velocity, or displacement. In the design case the standard of performance, which is minimum cost, is not known at the start; it has to be found by solving the problem, since it is the end result.

As we shall later see, this last difference eliminates the application, to the ship design problem, of the kind of optimization criteria used in the control problem. Other fields, such as operations research, have developed the so-called "dynamic programming" for performance optimization. Here again, there are some basic differences between the particular problems that this method was developed for and the ship design problem. Various aspects of the dynamic programming, closed form maxima and minima, as well as the steepest ascent techniques, are discussed in Appendix A. On the basis of the examination of each of these techniques, they were discarded and the exponential random search technique was chosen as most appropriate for the ship design process. This is discussed in the next section.

RANDOM SEARCH TECHNIQUES

A search of the literature reveals that most of the formal methods proposed for finding the maximum or minimum of a function are really useful only in the case of particularly simple functions. All of the methods are sensitive to a lack of continuity in the function or its derivatives, or to noise-type variations in the evaluation of the function. To use these methods for general problems, they must be combined with some sort of search procedure.

Although random search techniques are not new, their useful application to engineering problems is quite recent. Karnopp [3], in his doctoral thesis in 1961, pointed out some of the advantages in employing random search techniques as opposed to purely deterministic methods. Gall [2], in 1964, looked into utilizing various random search techniques for controlling a submarine in a random sea. Since as it was pointed previously, the design problem is different in its nature from a control problem, a basic re-evaluation of the optimization method has to be performed with the design problem in mind.

All search techniques are random, but some are more random than others. Even the most deterministic method can be classified as random with the randomness reduced to a minimum. Therefore, we have two extremes--on the one side, the purely random techniques and on the other, the purely deterministic ones. The notion that a probability of unity implies certainty and a probability of zero implies impossibility helps describe a deterministic technique. Deterministic schemes assume that the probability that a certain value of a function is the minimum is unity and that the probability that any other value of the function is the minimum is zero. In the case of the pure random search, the probability that the $(n + 1)^{st}$ trial point calculated will result in a lower value of the function being investigated is the same as the probability that the n^{th} trial point results in a lower value of the function. That is to say, the probability density function of a pure random search is constant as shown in Fig. 3, while the probability density function of a deterministic procedure would be a spike.

A pure random search is a powerful but not necessarily efficient procedure. It is powerful because simply by increasing the number of trial points calculated by the procedure, the probability that the procedure will calculate a trial point close to the precise location of the minimum value of the function, F_{min}, increases. But for this very reason, final convergence of a pure random search may be quite slow. That is, its "efficiency" is low. Hence, increasing the randomness of a search technique decreases its "efficiency". This is shown diagramatically in Fig. 1.

The concept of randomness is related to the concept of universality. That is to say, the more random a search procedure is, the greater is the number of kinds of functions to which that procedure is applicable. Therefore, when one does not know too much about a function which is to be searched, one wants initially a fairly universal method. As one learns more about the function being examined through actual application of a search technique, it would be desirable to be able to reduce the randomness of the search procedure so that it becomes more efficient.

If we plot the various search methods discussed in the introduction along the abscissa of Fig. 1, we see, for example, that Dynamic Programming is the most "efficient", and hence the least universal of the methods listed. The degree of universality of most of the methods cannot be adjusted, and they fall at fixed locations on this diagram. Only with the Exponential Random Search suggested by Gall can the degree of randomness, and hence universality, be adjusted. Thus, the user can change the randomness of the procedure as more is learned about the function from that shown in Fig. 3 to that of Fig. 4 to that of a strictly deterministic procedure. Hence, in Fig. 1, the Exponential Random Search extends across the entire abscissa. It is this feature of this technique that makes it so attractive.

There is a question with the Exponential Random Search procedure as to how much the probability density function of the random search should be influenced by the results of past trials. This question is a very important one because it differentiates two major classes of problems. One class of problems (which is probably the most common one), is where the function to be optimized can be evaluated in a matter of milliseconds on a high-speed digital computer, and the other class of problems is where the function's evaluation is very cumbersome because of the sophisticated, interdependent constraints and boundary conditions to which the function is subject.

In the case that we are faced with a problem of the first class, it will be more efficient to try a new point instead of setting up a sophisticated method to evaluate and utilize all the information from the past trials. In the second kind of problem, because of the large amount of time involved in evaluation of the function, it will be more efficient to utilize all information resulting from past trials and project ahead to help in choosing the next trial point.

In contrast to the various methods suggested by Karnopp [3], the search should have built-in tests for convergence and some self-optimizing features. Gall [2] suggested the exponential search which will be examined here and compared with some other functions that might give results similar in character, but with varying efficiency depending on the application.

In order to be able to analyze more rigorously the merits of one form of random search against another, it will be best to choose a simple function, for example, Fig. 2. It is required that we obtain the minimum of the function between the limits of $+K_a$ and $-K_a$. The function is:

1. Symmetrical about $K = 0$.
2. Only the portion $K = -K_a$ and $K = +K_a$ will be considered.
3. The function between $K = 0$ and $K = +K_a$ is monotonically increasing.
4. The function need not be continuous in slope.

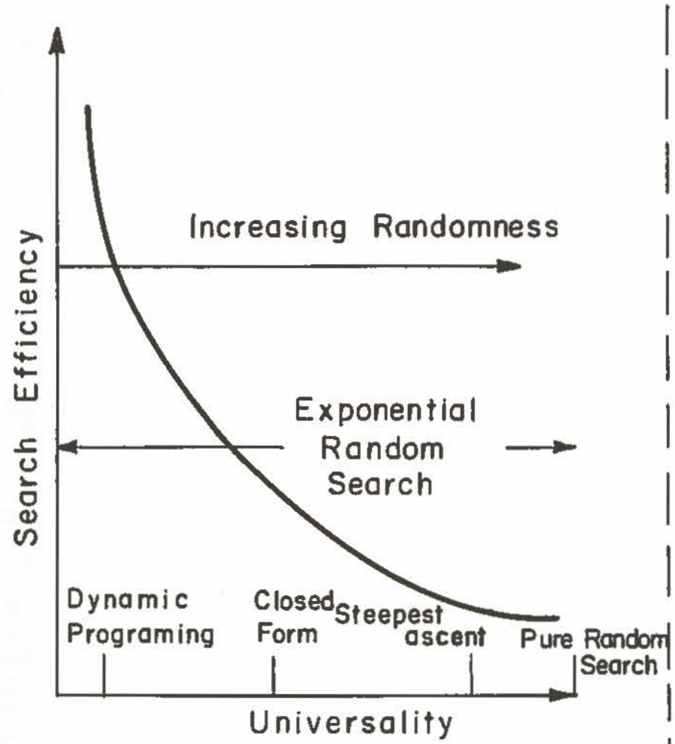

Fig. 1 Search Efficiency vs Universality

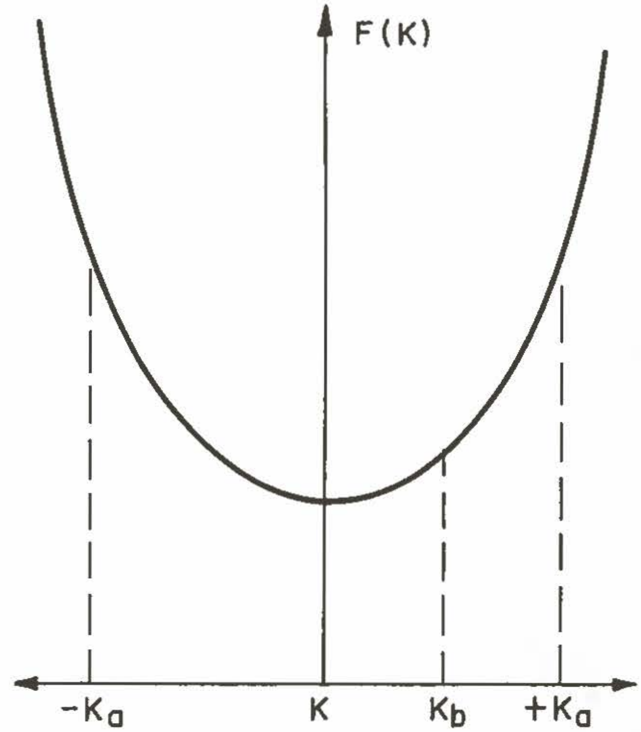

Fig. 2 One Dimensional Sample Function

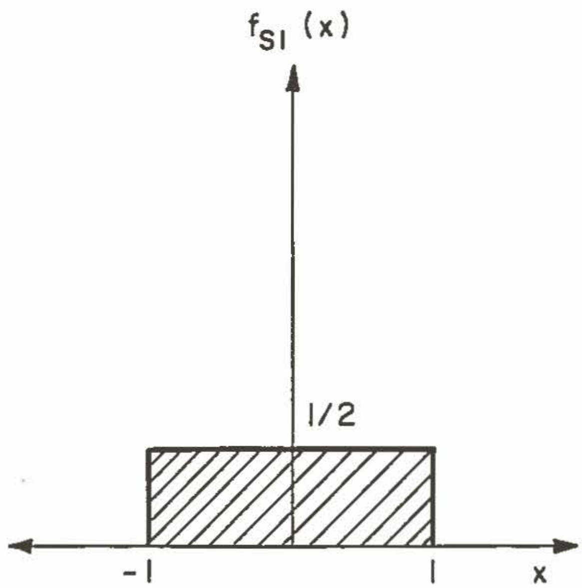

Fig. 3 Pure Random Search Probability Density Function

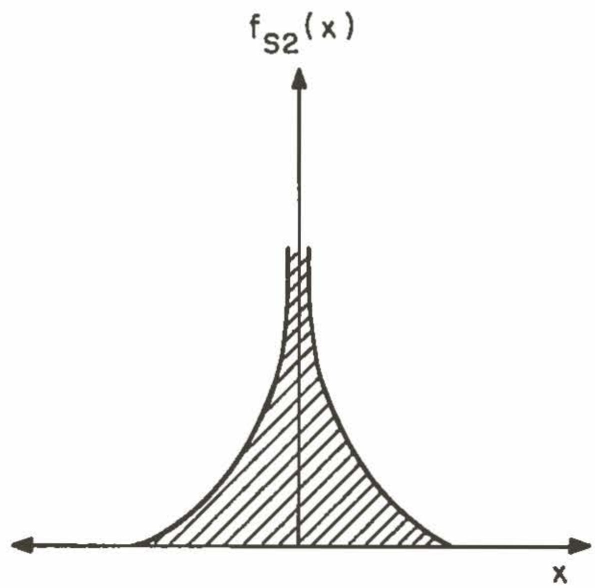

Fig. 4 Transformed Random Search Probability Density Function

After an arbitrary selection of the initial values in each search dimension has been made, a function that will generate the further trial points is necessary. This function will contain a random number, which is easily generated and is available as a library function in most computation centers. However this random number by itself is not sufficient since it will result in the complete random search procedure since its search probability density function is:

$$f_s(K) = \frac{1}{+K_a-(-K_a)} = \frac{1}{2K_a} \quad \text{for } |K| = K_a$$

and

$$f_s(K) = 0 \text{ elsewhere} \tag{3}$$

Since the probability is the integral of the density function, the probability of choosing a number within $\pm\Delta K_c$ units of the actual minimum, K_c, is

$$\underline{P}\ (K_c + \Delta K_c \geq K_c \geq K_c - \Delta K_c) = \frac{2\Delta K_c}{2\Delta K_a} = \frac{\Delta K_c}{K_a} \tag{4}$$

This is the least efficient random search, because any choice is equally probable, i.e. its probability distribution between $-K_a$ and $+K_a$ is constant, (see Figure (3)).

It is necessary to transform this purely random search to another more efficient one like that shown in Figure (4). Gall did this for an exponential case as follows:

$$K = 2K_a \, (G_K)^m + K_b \tag{5}$$

where G_K is the random number between 0 and 1. However, one may wonder why this particular formulation was selected and not for example:

$$K = 2\,K_a \ln(G_K) + K_b$$

or

$$K = 2\,K_a \cosh\,(G_K) + K_b$$

or

$$K = 2\,K_a \, e^{(G_K)} + K_b \tag{6}$$

or others. With every one of these functions, there exists an associated probability density distribution, a probability of improvement and a best expected step change. Although each one can be computed and compared, generalization of these results can put one on dangerous grounds.

There is no way to generalize results obtained from one function to another, (here function refers to the one to be minimized), and it is almost impossible also to classify these functions in various classes. Therefore, the attempt here is merely to gain insight and to check whether one of equations (5) or (6) is preferable to another.

Since we know that

$$f_s(G_K) = \frac{1}{2} \quad \text{for } (G_K) \leq 1$$

and

$$f_s(G_K) = 0 \quad \text{elsewhere} \tag{7}$$

This probability can be transformed for each of the equations (6) to find the probability density function for them.

It can be seen that each transformation above will give a function that has a varying probability density function. However, the only way that this probability function can be changed to give a higher and higher probability of improvement is to have a changing exponent as in equation (5). By using any of the other functions proposed in equation (6), only the first departure from a constant probability density function is achieved. The same is also achieved by a low powered exponent in equation (5). Therefore, it is concluded that the simplest way of accomplishing the objective is by use of the exponential random search, equation (5).

The transformation is done as follows:

$$K = 2\,K_a\,(G_K)^m + K_b \tag{8a}$$

or

$$K = a(x)^m + b \tag{8b}$$

since

$$f_s(G_K) = f(x) = \frac{1}{2} \qquad X_1 \leq X \leq X_2 \tag{9}$$

and

$$f_s(G_K) = 0 \qquad \text{elsewhere}$$

also from (8b) with $-1 < X < 1$

$$K_1 = a(-1)^m + b$$

$$K_2 = a(+1)^m + b \tag{10}$$

since by definition

$$\underline{P}\,(-1 \leq X \leq X_o) = \int_{-1}^{X_o} f(x)dx$$

also

$$\underline{P}\,(K_1 \leq K \leq K) = \int_{K_1}^{K = ax^m + b} f_s(K)dK \qquad (11)$$

also

$$\frac{\partial P}{\partial K} = f_s(K) = \frac{\partial}{\partial K} \int_{-1}^{x = \left[(K-b)/_a \right]^{1/m}} f_s(x)dx$$

since

$$f_s(K) = \frac{\partial \int f_x(x)}{\partial K} \frac{\partial X}{\partial X} = \frac{\partial \int f_s(x)}{\partial X} \frac{\partial X}{\partial K}$$

and since; $a = 2\,K_a$, $b = K_b$, and $f_s(x) = \frac{1}{2}$, (12a)

$$f_s(K) = \frac{1}{4\,K_a m} \left(\frac{K - K_b}{2\,K_a} \right)^{(1-m)/m}$$

If the function F(K) is not symmetrical then the limits can be changed from $(-K_a \rightarrow +K_a)$ to $(K_a \rightarrow K_d)$ and the probability density function will be

$$f_s(K) = \frac{1}{2m(K_a - K_d)} \left(\frac{K - K_b}{K_a - K_d} \right)^{(1-m)/m} \qquad (13)$$

which is the general form of equation (12).

Using equation (13) for the probability density function the probability of improvement* and the expected change step follows by definition:

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* Meaning the probability that $(K_{b_{i+1}} < K_{b_i})$ subject to the condition that it is in the range of $\pm\,K_a$.

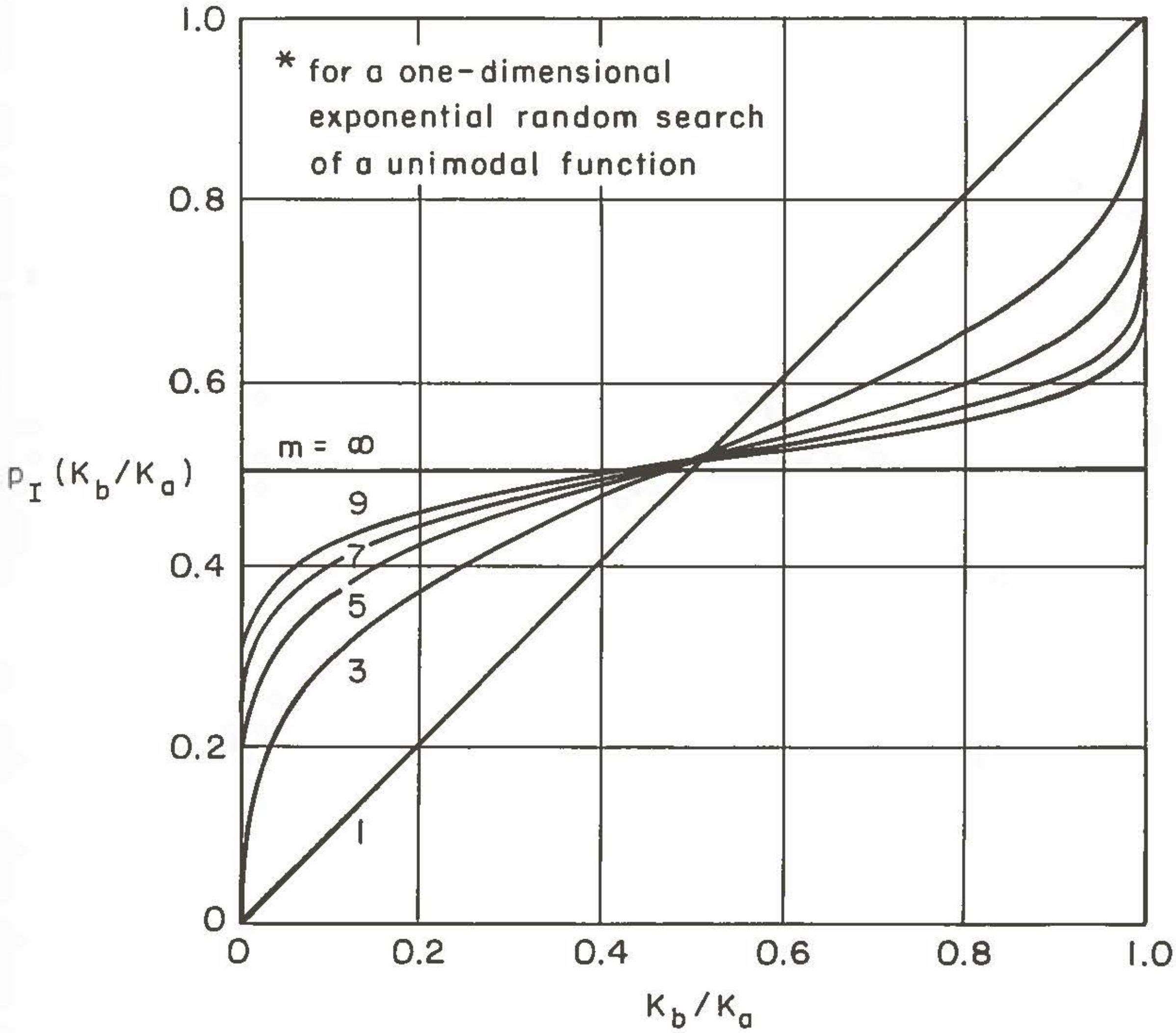

Figure 5. Probability of Improvement *

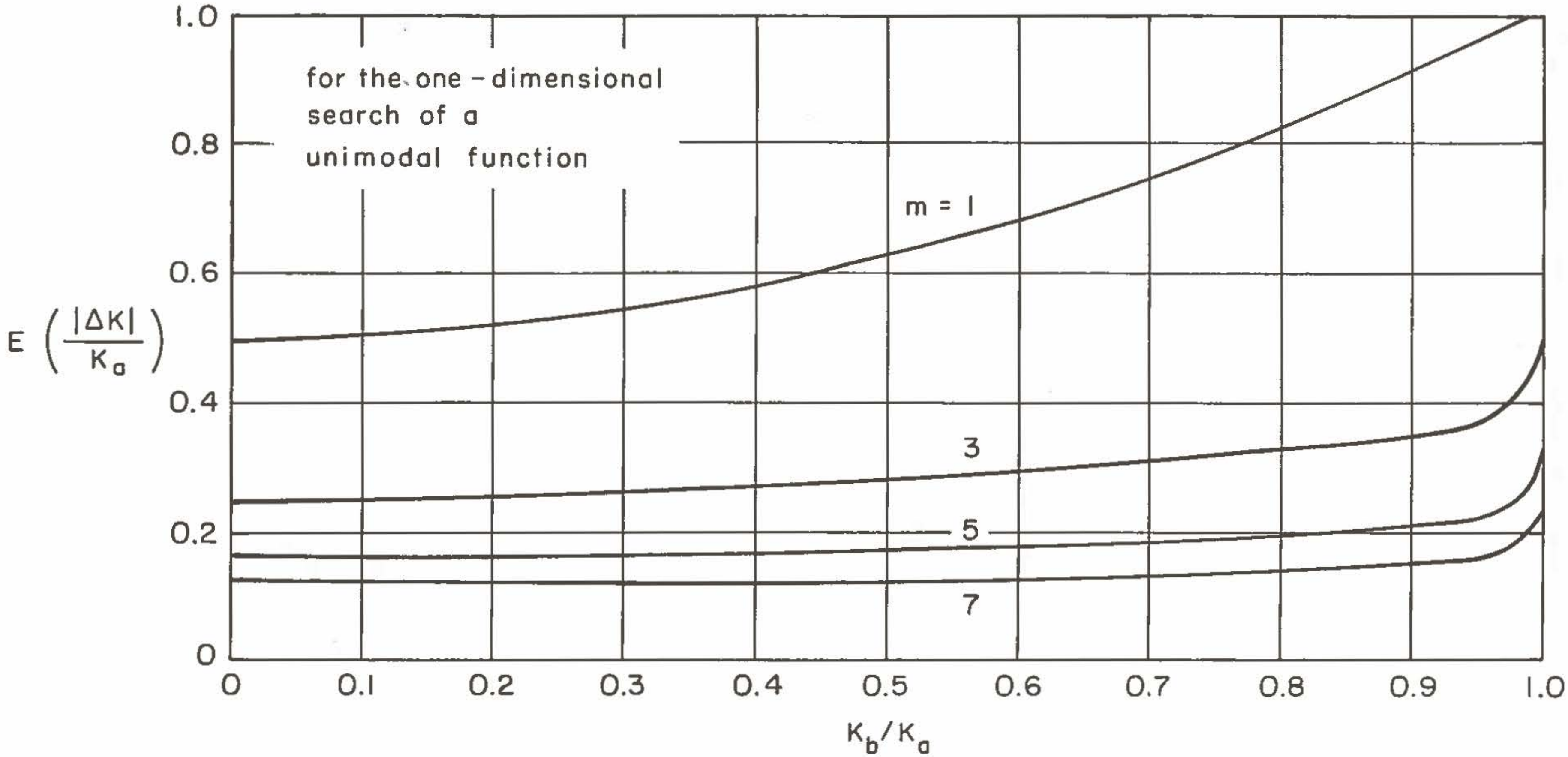

Figure 6. Expected Step Length*

$$\underline{P_I}(K_b) = \frac{\int_{-K_b}^{+K_b} f_s(K)dK}{\int_{-K_a}^{+K_a} f_s(K)dK} = \frac{(K_b/K_a)^{1/m}}{\left(\frac{1+K_b/K_a}{2}\right)^{1/m} + \left(\frac{1-K_b/K_a}{2}\right)^{1/m}} \tag{14}$$

$$E\left(\frac{|\Delta K|}{K_a}\right) = \frac{\int_{-K_b}^{+K_b} \frac{K-K_b}{K_a} f_s(K)dK}{\int_{-K_a}^{+K_a} f_s(K)dK} = \frac{2}{m+1} \frac{\left(\frac{1-K_b/K_a}{2}\right)^{\frac{m+1}{m}} + \left(\frac{1+K_b/K_a}{2}\right)^{\frac{m+1}{m}}}{\left(\frac{1-K_b/K_a}{2}\right)^{1/m} + \left(\frac{1+K_b/K_a}{2}\right)^{1/m}} \tag{15}$$

These results from Gall are shown on Figures (5 and 6).

So far only a one-dimensional (n=1) search has been examined. Since the search in one variable space is independent of the search in another, the joint probability distribution is the product of the individual probability density functions. This is:

$$f_s(K^{(1)}, K^{(2)} \ldots K^{(n)}) = \left(\frac{1}{2m(K_a-K_d)}\right)^n \left(\frac{K-K_b}{K_a-K_d}\right)^{\left[(1-m)^n/m\right]^n} \tag{16}$$

Again, the probability of improvement can be computed as before and plotted as done in Figures (7, 8) for two different exponents and several dimensions.

The one-dimensional search (n=1) is also plotted since it represents the maximum search efficiency. From Figures (7 and 8) it is evident that the efficiency of search decreases rapidly as a function

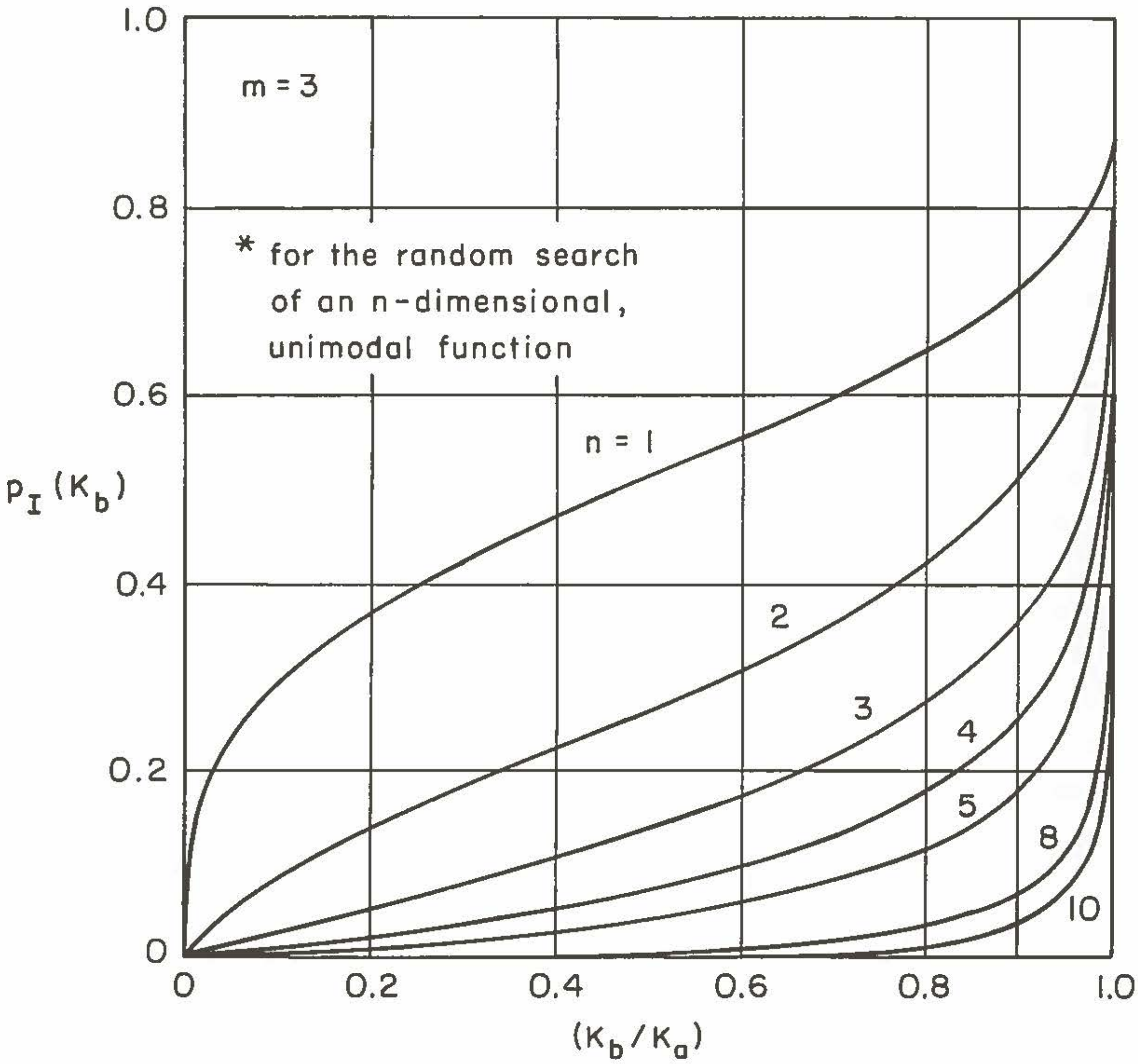

Figure 7. Probability of Improvement*

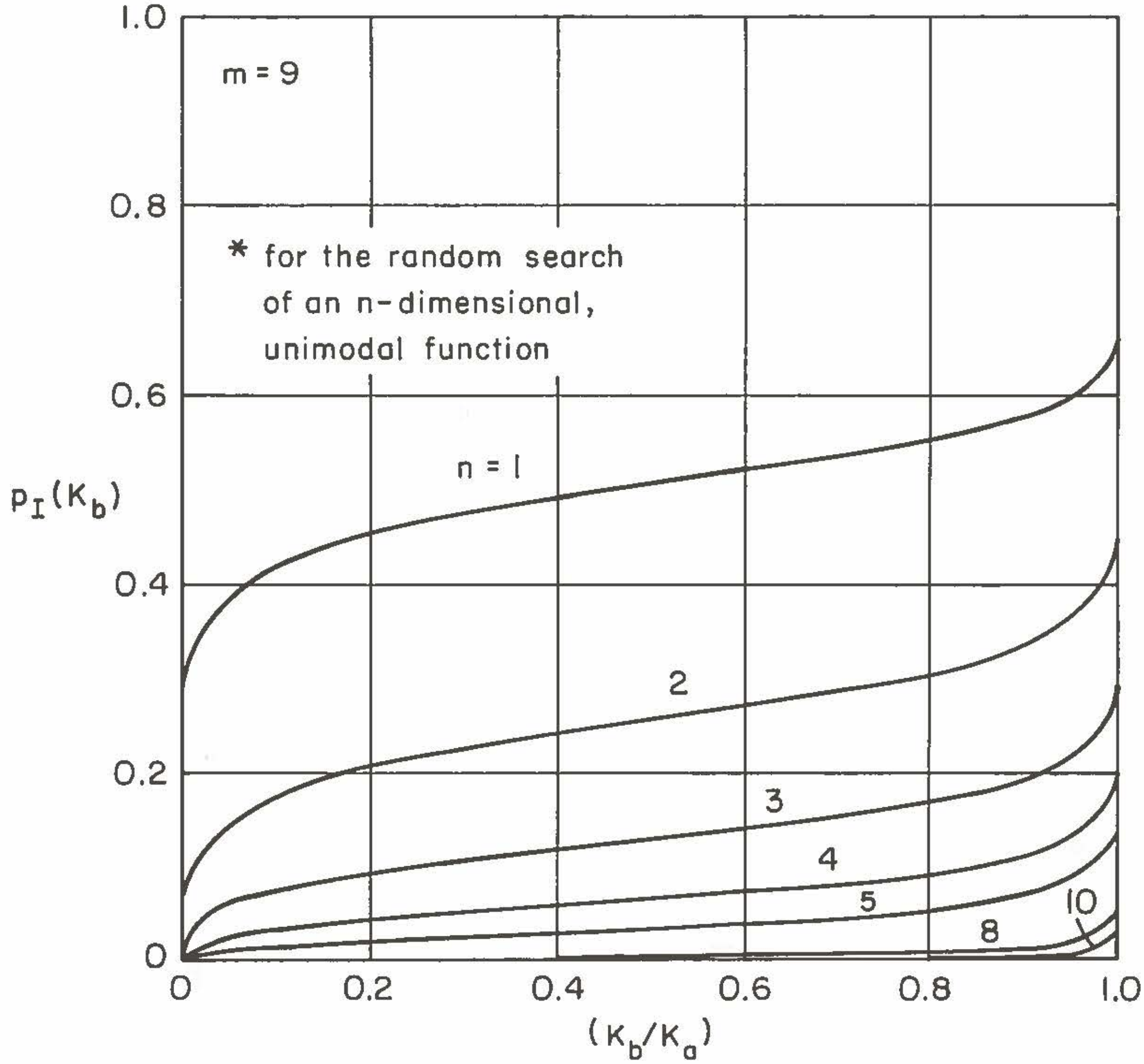

Figure 8. Probability of Improvement *

of n. An empirical relation is quoted by Gall [2].

$$(N_c)_n \approx (N_c)_1 \, 2^{n-1} \tag{17}$$

where N_c is the number of choices and n the dimension of the search space. The number of choices refers to the ones which do not fall outside the bounded region.

The random search as explained here is very easily computerized; it is efficient, general and fast. Without the necessity of using derivatives, the process behaves much like a gradient method. The random process has the advantage of simplicity, insensitivity to discontinuities in slope and has an easily adjustable parameter which can be changed as the search process proceeds, in order to accelerate the rate of convergence. The random methods that will be analyzed here can be tested by means of mental or mathematical experiments. Hence, it would be unwise to conduct extensive computational experiments.

Its drawbacks are:

1. It suffers from dimensionality i.e. the time that it takes to converge will increase very fast as a function of the space dimensions.

2. In the case of the design problem, it is time consuming to evaluate the function many many times, whereas this was not the case in the control problem studied by Gall.

Given a general function, the probability of success for either the random search or the presently used brute force approach (Ref. 1) is the same. However, the utilization of a systematized random search such as the exponential one is far superior to the "brute force" method. While the latter method marches through its search space at equal intervals and then marches through again at some finer intervals to converge on a minimum, the systematized random search optimizes itself always around its best test result. This follows from the fact that the systematized random search does not have a uniform probability density function such as the purely random and the "brute force" methods have. At first, the random search is started by a low exponent Equ. (13) so it encompasses a large search space with relatively low probability of improvement. Once it has started concentrating about some area, a higher exponential search is invoked, thus increasing the probability of improvement at the expense of reducing the step size.

This process of changing the exponent is continued until the method becomes actually a refinement rather than a search. It terminates itself by exceeding a preset tolerance. Thus, even though it starts out as a purely random search with equivalent probability of improvement to the equal interval step method, it rapidly improves on itself as the search for the optimum progresses.

The method itself is very simple to apply. When the exponent equals one, the search is purely random. If the function is nonsymmetrical as is the case of Equ. (13), the searching function in general is:

$$K = (K_d - K_a)(2G_K - 1)^m + K_b \tag{18}$$

where G_K is the random number,

K_b is the best last search point,

K_d and K_a are the limits of the search space

and K is the new trial point.

For a two-dimensional case the process is repeated twice and for an n dimensional case n times.

OPTIMIZATION CRITERIA

A very important step in the optimization process is a proper choice of the optimization criteria, since it determines the final outcome of the process. While control optimization or allocation optimization to name only two, can utilize various performance criteria, the design optimization is of different nature and more limited in choice. Let us examine three out of a large variety of possible optimization criteria.

Mean Square Error Criterion

This criterion is the simplest both conceptionally and mathematically. The two most common and most easily determined statistical quantities are the mean and the variance (or mean square). The product xy of two random variables is a random variable which equals $x_i y_j$ when $x = y_j$. Although it is not usually true that the expectation of a product is the product of the expectations, this is the case when the variables are independent.

$$E(xy) = E(x)E(y) \qquad x,y \text{ independent} \tag{19}$$

(The proof can be found in any book on the subject).
When the problem involves several variables x,y.....it is convenient to denote the expectation by a different letter. Let us say

$$E(x) = \mu_x \tag{20}$$

To measure the deviation of a variable from its expected value μ, one introduces a quantity σ defined by:

$$= \sqrt{E(x-\mu)^2} \quad \text{or} \quad \sigma^2 = E\left[(x-\mu)\right]^2 \tag{21}$$

The expression σ is called standard deviation and its square σ^2 is called the variance or (mean square).

Multiple-Parameter Weighting Criterion

Based on the same principal as the previous criterion, but with added generality, the multiple-parameter weighting criterion is more flexible. It allows for weighting each parameter against each other, thus increasing or decreasing the role of each parameter in the optimization process at will

$$C = W_1\ (x-\mu_x)^2 + W_2\ (x-\mu_y)^2 + W_3\ (z-\mu_z)^2 + \ldots \tag{22}$$

Multiple-Parameter Max-Ranking Criteria

This method employs a ranking array. The essential factor here is that each system attribut which is to be considered in the optimization is rated against an absolute scale of desirability. An example of a possible ranking array for a system of 4 variables is given in Table I. For example, consider row 2 $C(I) = 1$. A value of $DEL_1 = 3$ is considered as desirable an end result as $DEL_2 = 8$ or $DEL_3 = 0.7$.

Desirability	C(I)	DEL_1	DEL_2	DEL_3	DEL_4
	0	0	1	0	0
Most Desirable	1	3	8	0.7	0.2
	2	4	10	0.9	0.3
Least Desirable	3	4	17	1.0	0.3
	4	9	18	1.2	0.5

TABLE 1 MAX-RANKING ARRAY

The construction of this ranking array should be carried out with a great deal of thought. The results of the entire optimization study will depend upon the values selected. In order to construct the ranking array, the designer must have a good appreciation of the system capabilities and requirements.

Once the ranking array is set up, it can be applied in a straight forward simple manner. The first step is to assign values to each C(I) for any given set of system variables. This can be accomplished by any interpolation scheme which the designer desires to employ.

As an example, let us refer back to TABLE 1. Let us suppose that for a given ship design the difference between the desired payload and the actual payload is $DEL_1 = 4$ and the difference between the desired stowage factor and actual stowage factor is $DEL_2 = 9$. $DEL_3 = 1.0$ and $DEL_4 = .4$ are some other pertinent parameters. Then linear interpolation in TABLE 1 gives the following values for each C(I).

C(1) = 2.0
C(2) = 1.5
C(3) = 3.0
C(4) = 3.5

This essentially establishes the desirability (for this particular case) of each of the four resulting variables.

Two different approaches can be taken in an attempt at assigning an overall desirability based upon several values of the individual C(I)'s.

The first method defines the overall desirability as the average desirability of the resulting individual variables. For the particular example being considered, this gives:

$$C_s = \frac{1}{4} \sum_{x=1}^{4} C(I) = \frac{1}{4} (2.0 + 1.5 + 3.5) = 2.5 \tag{23}$$

This method is exactly equivalent to the multiple-parameter weighting criterion, whose weighting parameters are functions determined by the ranking array. This method has only one advantage over the weighting method, i.e., a methodology for determing the weighting functions. The principle disadvantage is that there is implicit weighting between the columns of the ranking array. The minimum value of the system desirability (C_s) based upon this averaging method would be one which produced low values of C(1), C(2) and C(3) at the expense of higher values of C(4).

A much better method is one which equates the overall system desirability with the value of C(I) corresponding to the <u>least</u> desirable of the resulting individual attributes of the system. This, in effect, states that the system is no more desirable than its least desirable attribute. This method has been termed Max-Ranking. The Max-Ranking measure of system desirability (C_M) is defined simply as:

$$C_M = C(I)_{max} \tag{24}$$

Thus, for the example being used here, C_M is

$$C_M = C(1) = 2.0;\ C(2) = 1.5;\ C(3) = 3.0;\ C(4) = 3.5$$

or

$$C_M = C(4) = 3.5$$

The criterion of optimality is, that C_M should be minimized.

SELECTION OF AN OPTIMIZATION CRITERION

On the basis of an evaluation of the ship design process, it was decided that the overall optimization criterion should be composed of three terms. One of these is an economic criterion and following Ref. (1) this has been chosen to be the sum of yearly operating costs plus annual depreciation and interest charges. Clearly the optimization process should seek to minimize this cost rather than seek to achieve a certain predetermined level of this cost. Unfortunately, this fact eliminates the Max-Ranking criterion as a possible optimization criterion for the ship design process, since as noted in the previous section, the Max-Ranking criterion can deal only with optimization noted against an absolute scale of desirability.

The other two parts of the optimization criterion constitute the boundary conditions of the criterion. They are two owner's requirements; payload weight and payload volume (stowage factor)*. These two factors are compatible with the Max-Ranking criterion since a prescribed value of those two factors is sought in the design process. However, even for these two, setting up an absolute scale of desirability can only be done in an artificial way.

However, because of the fact that use of the Max-Ranking criterion is incompatible with the least cost criterion, the multiple parameter weighting criterion was chosen as the best suited to the ship design problem. Following the previous discussion and equation (22),

*The other two owner's requirements, speed and range are assumed as fixed input items.

this criterion is:

$$C = W_1 * (DELWP)^2 + W_2 * (DELSF)^2 + W_3 * (COST)^2 \qquad (26)$$

where

W_1, W_2, and W_3 are weighting factors.

DELSF - is the difference between the actual and required stowage factors

DELWP - is the difference between the actual and required payload weight

COST - straight line depreciation plus average interest, 25 year economic life, 2.5% scrap value, 5% simple interest, reduced to a yearly expenditure basis. A yearly fuel cost is then added to this figure.

THE MATHEMATICAL MODEL

What is needed in the ship design problem is the mathematical model that determines the cost (initial plus operating costs) as a function of the basic parameters defining the form and size of a ship. These basic parameters are listed below along with the major technical considerations and weight groups which they influence. Also listed are the two functional interrelationships between the displacement and the weight groups and between the displacement and the other basic ship form parameters.

(1) Length, L - power, (W_m, W_f, W_s)

(2) Breadth, B, - stability and period of roll, (W_s)

(3) Depth, D, - strength, (W_s)

(4) Draft, H, - hydrographic restrictions

(5) C_p - residuary resistance, (W_m, W_f)

(6) Δ - displacement, $\Delta = W_s + W_m + W_f + \ldots\ldots + W_p$ and

$$\Delta = \frac{C_p \times C_m \times L \times B \times H}{35}$$

where

W_m - machinery weight group

W_f - fuel weight group

W_s - structure weight group

W_p - payload weight group

Because the time required to obtain a solution is affected adversely by the number of dimensions of the search space, it is essential to minimize the number of variables. One way of accomplishing this is to combine the ship dimensions into dimensionless ratios. Thus the following four dimensionless and one-dimensional parameters can replace the previous six-dimensional parameters.

1. B/H
2. L/D
3. $V/\sqrt{gL}$ or $V/\sqrt{L}$ $(C_m = f(V/\sqrt{L})$
4. C_p
5. Δ

One of the variables must remain dimensional and this was chosen to be Δ. But it is clear that it could also have been B or H or L, since Δ = $LBHC_pC_m$. The third parameter will be recognized as the Froude number or speed-length ratio. Since in this parameter, speed is a prescribed input item, an initial random selection of $V/\sqrt{L}$ fixes the initial length selection.

The optimization criterion C, of Eq. (26) will be a function of the preceding 5 variables, i.e.,

$$C = f\ (\Delta,\ B/H,\ L/D,\ V/\sqrt{L},\ C_p) \tag{27}$$

To completely define this function, the midship section coefficient is required too. Because it has only a small influence on the final result, it was assumed to be a dependent variable, i.e., $C_m = f(V/\sqrt{L})$. (see Equation 1 of Appendix B). The criterion C is also subject to two additional constraints.

1. Load line regulations require a certain minimum freeboard for each ship as a function of L mainly, but also depending on D and C_B. ($C_B = C_p \times C_m$). In this study the relatively small dependency on D and C_B was neglected.

2. The second constraint is a minimum stability requirement which is expressed in terms of minimum acceptable GM/B ratio.

A question exists as to why displacement is selected as a basic dimension of the search space. For example, it can be argued on theoretical grounds that the specification of the four owner's requirements, i.e. speed, range, payload weight and stowage factor along with an initial random selection of the four basic dimensionless parameters, B/H, L/D, $V/\sqrt{L}$ and C_p determine a unique value of displacement. While this is true theoretically, practically, there is great difficulty in determining this value of the displacement primarily because the estimates of ship power require preknowledge of ship displacement as well as ship speed and the other ship dimensions and coefficients. For this reason, in the current work, displacement is one of the basic dimensions of the search space. Thus for each initial selection of displacement and the other four basic dimensionless parameters, plus the initial input of a required ship speed and range, there exists a unique value of payload weight and of stowage factor. The selected optimization criterion then seeks to minimize the error between these values of payload weight and stowage factor and the values specified by the owner as well as to minimize cost. In this way, the random search method eventually achieves compatibility amongst all of the parameters involved in the ship design process.

THE SOLUTION

The ranges of the five basic parameters within which the random search process will conduct its search are determined as follows:

(1) B/H, $V/\sqrt{L}$, & C_p: The ranges of these three parameters are determined by the coverage of the model resistance series used to determine residuary resistance. If the Taylor's standard series (ref. 5) is used, the ranges are as follows:

B/H - 2.25 to 3.75

$V/\sqrt{L}$ - 0.5 to 2.00

C_p - 0.48 to 0.80

These represent far broader ranges than are likely to be needed for most conventional ship designs.

(2) L/D: The upper limit of this parameter is restricted by the minimum permissable freeboard specified by the U.S.C.G. Load Line Regulations and given in Equations 15, 16, 17 of Appendix B. (See also Eq. 14). For the current study, no lower limit was placed on the range of this parameter.

(3) Δ: The range of this parameter is initially selected arbitrarily by the designer based on his experience. If his initial selection of the displacement range is poor, this will be immediately evident as the random search progresses because there will be a persistently large error, either always positive or always negative between the required values of payload weight and stowage factor and the values computed by the program. If this happens, the displacement

range can be readily adjusted by the designer either upward or downward after several trial points are generated.

The optimization criterion C, is then evaluated starting with randomly selected initial values (within the ranges of values just discussed) for each of the five parameters, (B/H, L/D, $V/\sqrt{L}$, C_p, Δ). Based on these initially selected values, the program carries out computation of the following items sequentially:

a. The frictional resistance coefficient using the ITTC line.

b. The residuary resistance coefficient using Taylor's Standard Series, Ref. (5).

c. Knowing the total resistance and hence the total power (see Appendix B, Item C,) as well as the ship dimensions, W_m, W_s, W_f, etc. are computed.

d. The total cubic space available.

e. The total cost.

f. The two constraints, stowage factor and payload weight.

For the computation of the various weights, centers of gravity, stability (GM) and costs, empirical equations and constants derived from Ref. (1) and listed in Appendix B are used. These empirical relationship, which are not universally applicable to all types of ships, can be changed without affecting the general method which is tested here. To make the computer program completely general, the empirical expressions in their algebraic form could have been read in to the computer as data using a Fortran compatible language called FORMAC. This, however, was not actually done in this study.

Once the optimization criterion C is calculated for the initially selected random values of the five dimensions, the optimization procedure can commence. A new random value is generated in each search dimension, a new C is computed and compared to the previous value. Only values of C smaller than previous ones are used to generate further values. Larger values are discarded.

The procedure, thus far, results in repeated evaluations of C within the coverage of the five-dimensional space function. As explained in the section describing the random search technique, increasing the exponent of search, results in a decrease of the expected step length on the account of the increase of probability of improvement. Thus gradually, the exponent is increased, starting with n = 1 (pure random search) so as to cover the whole search space at the beginning, to larger exponents in order to increase search efficiency. The search procedure is terminated after a prescribed number of good choices fails to produce an improvement on a preset tolerance.

RESULTS AND CONCLUSIONS

In order to demonstrate the correctness of the results of the design subprogram developed in this study, the characteristics of the sample ship shown in Ref. (1) on page 66 were reporduced by using the same input parameters Δ, C_p, $V/\sqrt{L}$, B/H and L/D as in Ref. (1). These results, together with those computed in Ref. (1), are shown in TABLE II. As it can be seen, agreement is excellent. The slight difference in the cost and payload weight stems from the slightly higher horsepower which, in turn, results in slightly higher fuel weight and machinery weight. This, in turn, reduces slightly the payload weight since the displacement is the same. However, all of these slight differences which may result from different ways of interpolating the Taylor's standard series are within an error of $\pm$ 0.5%.

Having shown that the design subprogram of the optimization method produces accurate results, there remains to be demonstrated by actual calculations, that the optimization method developed in this study has merit compared to current computational as well as manual techniques. It is believed that the proposed method has the following advantages:

(1) It searches out lower cost ship designs for a given set of owner's requirements.

(2) If there is more than one lowest cost design, the method will locate it.

TABLE II

VERIFICATION OF THE RESULTS OF THE SHIP DESIGN PROCESS

Design No. Source	(1) Design From Random Search Closest to Ref. (1)	(2) Design From Ref. (1)		(1) Design From Random Search	(2) Design From Ref. (1)
Range	13000.	13000.	L	509.6	509.5
Speed	20.0	20.0	B	80.3	80.3
Payload	8977.33	9001.52	H	29.3	29.3
			SHP	16904.	16775.
S.F.	90.95	90.84			
Cost Points	271124	2700386			
Displacement	19891	19891	WO	2214.6	2214.6
C_p	.597	.597	WS	4668.7	4669.0
$V/\sqrt{L}$	.886	.886	WM	889.2	885.8
B/H	2.74	2.74	WF	2841.2	2820.6
L/D	10.48	10.47	Margin	300.0	300.0

(3) It performs the necessary calculations more quickly and at less cost than current methods.

(4) It is more flexible and more versatile than any other method available.

(5) As a result of having the computer program written for a time-sharing system, the proposed method permits continuous dialogue between the designer and the computer.

(6) By utilizing a newly developed computer language called FORMAC, all the empirical expressions in their algebraic form can be input to the program rather than part of it, thus enabling quick and easy changes of the empirical information without the necessity of altering the program itself.

The first advantage is demonstrated in TABLE III which shows a comparison between the characteristics of two ship designs computed by different methods, but all intended to conform to a range of 13,000 miles, a speed of 20 knots, a payload weight of 9,000 tons and a stowage factor of 90. It is clear that the proposed method seeks out a design that conforms more closely to the owner's requirements and for less cost than the computational approach of Ref. (1).

The reason for this result is the fact that the current method treats the optimization problem as a five-dimensional surface, rather than breaking it down into five one-dimensional curves as is done in Ref. (1). By constructing a smooth curve through four points, small variations in the cost curve may have been washed out in Ref. (1).

TABLE III

COMPARISON OF SHIP DESIGNS COMPUTED BY TWO METHODS

Design No.	(4) Computer Design from Ref. (1)	(3) Minimum Cost By Random Search		(4) Computer Design from Ref. (1)	(3) Minimum Cost By Random Search
Range	13000	13000	L	518.2	534.6
Speed	20.0	20.0	B	80.23	77.4
Payload	9002 (owner's req't.)	9005.7	H	29.93	28.76
SF	89.77	88.47	D	48.54	46.6
Cost Points	277,884	269,489	GM/B	.0504	.0510
			SHP	16,372	16,277
Displacement	20,239	19,890	WO	2272	2255
C_p	.585	.600	WS	4756	4715
$V/\sqrt{L}$	.879	.865	WM	875	873
B/H	2.680	2.692	WF	3034	2741
L/D	10.680	11.500	Misc. Deadwgt.	300	300
			Margin	---	---

By treating the multi-dimensional surface as it really exists, small undulations are detectable. When searching for a minimum cost ship in a limited range of the variables, it is obvious that the cost variations are not going to be large. Rather, the hope is to find such a set of ship characteristics which would be slightly better than others in the specified range.

TABLE IV demonstrates the second advantage of the proposed method. Since a five-dimensional surface may be multimodal, there might be several combinations of ship characteristics which yield the same cost. A demonstration that there are at least two designs of quite different dimensions that are close to minimum cost ship is displayed in TABLE IV.

As far as the third advantage is concerned, the method proved to be very efficient. A normal search of 500 search points took about one minute on the IBM 7094 computer.

To demonstrate the flexibility and versatility of the method, it will be recalled that the criterion consists of three parts; the payload weight, the stowage factor and the cost, each associated with a particular weighting factor. In the design stage, the owner may also be interested in the effect of variations of the payload weight and stowage factor on cost and ship dimensions. The output of a random search for an optimum design will not only yield the least cost design, but will also disclose the effect of small changes in stowage factor and payload weight on cost. This flexibility is achieved very simply by weighting one factor more than another in the criterion. These effects are displayed in TABLE V.

Advantages five and six are a result of utilizing the latest advances in digital computer methods, software and systems.

TABLE IV

TWO SHIPS WITH IDENTICAL COST

Design No.	(5) Ship No. 1	(6) Ship No. 2		(5) Ship No. 1	(6) Ship No. 2
Range	13000.0	13000.0	L	506.6	534.8
			B	81.1	77.0
Speed	20.0	20.0	H	29.7	28.6
			D	48.4	46.5
Payload	8997.58	8989.94	GM/B	.054	.050
S.F.	88.879	88.874	SHP	16755.9	16592.9
Cost Points	270787.	270539.			
Displacement	19891.	19903	WO	2229.1	2239.4
C_p	0.587	0.606	WS	4661.6	4700.9
$V/\sqrt{L}$	.889	0.865	WM	885.3	881.0
B/H	2.731	2.692	WF	2817.6	2791.7
L/D	10.465	11.500	Margin	300.0	300.0

TABLE V

THE EFFECT OF WEIGHTING FACTORS

Design No.	(7)	(8)	(9)	(10)
W1	1.000	50.000	1.000	1.000
W2	1.000	1.000	50.000	1.000
W3	1.000	1.000	1.000	50.000
Required Payload	9000.0	9000.0	9000.0	9000.0
	8973.5	9000.2	9051.9	8859.0
Required Stowage Factor	90.0	90.0	90.0	90.0
Actual Stowage Factor	88.26	86.77	90.27	87.74
Cost Points	269,756.7	269,364.6	279,374.6	267,996.8
Δ	19887.6	19887.6	20379.9	19692.0
C_p	.584	.583	.566	.583
$V/\sqrt{L}$	.867	.867	.864	.867
B/H	2.714	2.700	2.822	2.700
L/D	11.419	11.500	11.441	11.500

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APPENDIX A.

THREE METHODS OF OPTIMIZATION

(a) Closed Form Solution

Given a function $f(x,y)$ where x and y are two independent variables, it can be shown that the necessary condition for a maximum (or minimum) of $f(x,y)$ at $x = a$ is that

$$\frac{\partial f}{\partial x} = 0 \tag{1-A}$$

if this derivative exists at $x = a$. Similarly, $f(x,y)$ will attain a maximum (or minimum) at $y = b$ if

$$\frac{\partial f}{\partial y} = 0 \tag{2-A}$$

and the derivative exists. The coordinates (a,b) thus satisfy the equations

$$\frac{\partial f}{\partial x} = 0 \text{ and } \frac{\partial f}{\partial y} = 0 \tag{3-A}$$

at any point (a,b) where $f(x,y)$ attains a maximum or minimum.

In addition, the problem of design will undoubtedly contain variables which are not independent, thus resulting in a constrained extremum.

Let us consider the following problem:

Given a function of several variables $f(x_1, x_2, x_3 \ldots x_n)$ subject to several constraints:

$$\begin{aligned} \varphi_1 (x_1, x_2, x_3 \ldots x_n) &= 0 \\ \varphi_2 (x_1, x_2, x_3 \ldots x_n) &= 0 \\ \\ \varphi_n (x_1, x_2, x_3 \ldots x_n) &= 0 \end{aligned} \tag{4-A}$$

and each variable bounded by some two values, what is the absolute extremum? The equivalent single requirement to the vanishing of $\frac{\partial f}{\partial x}$ and $\frac{\partial f}{\partial y}$ simultaneously, is the vanishing of its total differential at the maximum and minimum points of the function; i.e.,

$$\frac{\partial f}{\partial x_1} dx_1 + \frac{\partial f}{\partial x_2} dx_2 + \frac{\partial f}{\partial x_3} dx_3 \ldots \frac{\partial f}{\partial x_n} dx_n = 0$$

Equation (4-A) yields also the following:

$$\frac{\partial\varphi_1}{\partial x_1}\,dx_1 + \frac{\partial\varphi_1}{\partial x_2}\,dx_2 + \frac{\partial\varphi_1}{\partial x_3}\,dx_3 + \dots \frac{\partial\varphi_1}{\partial x_n}\,dx_n = 0$$

$$\frac{\partial\varphi_n}{\partial x_1}\,dx_1 + \frac{\partial\varphi_n}{\partial x_2}\,dx_2 + \frac{\partial\varphi_n}{\partial x_3}\,dx_3 + \dots \frac{\partial\varphi_n}{\partial x_n}\,dx_n = 0 \qquad (5\text{-A})$$

For purposes of simplicity and since by now the generality is obvious, let us consider that the function is only dependent on three variables, subject to two constraints; i.e.,

$$f(x,y,z) = 0$$
$$\varphi_1(x,y,z) = 0$$
$$\varphi_2(x,y,z) = 0$$

Using the method developed by Lagrange, the total differential of the constraints are multiplied and added to the equation of the function to obtain:

$$\left(\frac{\partial f}{\partial x} + \lambda_1\frac{\partial\varphi_1}{\partial x} + \lambda_2\frac{\partial\varphi_2}{\partial x}\right)dx + \left(\frac{\partial f}{\partial y} + \lambda_1\frac{\partial\varphi_1}{\partial y} + \lambda_2\frac{\partial\varphi_2}{\partial y}\right)dy$$

$$+\left(\frac{\partial f}{\partial z} + \lambda_1\frac{\partial\varphi_1}{\partial z} + \lambda_2\frac{\partial\varphi_2}{\partial z}\right)dz = 0 \qquad (6\text{-A})$$

Now, let λ_1 and λ_2 be determined so that two of the parentheses in Equation (6-A) vanish. Then, the differential of the constraints multiplying the remaining parenthesis can be arbitrarily assigned and that parenthesis must also vanish. In other words,

$$J(x,y) = \begin{vmatrix} \frac{\partial\varphi_1}{\partial x} & \frac{\partial\varphi_1}{\partial y} \\ \frac{\partial\varphi_2}{\partial x} & \frac{\partial\varphi_2}{\partial y} \end{vmatrix} \neq 0 \qquad (7\text{-A})$$

where J(x,y) is the Jacobian of x and y.

Thus, we must have:

$$\frac{\partial f}{\partial x} + \lambda_1 \frac{\partial \varphi_1}{\partial x} + \lambda_2 \frac{\partial \varphi_2}{\partial x} = 0$$

$$\frac{\partial f}{\partial y} + \lambda_1 \frac{\partial \varphi_1}{\partial y} + \lambda_2 \frac{\partial \varphi_2}{\partial y} = 0 \qquad (8\text{-A})$$

$$\frac{\partial f}{\partial z} + \lambda_1 \frac{\partial \varphi_1}{\partial z} + \lambda_2 \frac{\partial \varphi_2}{\partial z} = 0$$

$$\varphi_1 (x,y,z) = 0$$

$$\varphi_2 (x,y,z) = 0$$

The result is five equations with five unknowns--x, y, z, λ_1, and λ_2.

It can be shown that there will be a number of equations corresponding to the number of unknowns even if the constraints contain only part of the number of unknowns or if there are more constraints than unknowns or any other limiting case. Thus far, this closed form solution of the general n dimensional problem seems very promising for the design problem. There are, however, a number of drawbacks that will now be examined. As before, for simplicity, we will restrict ourselves to functions of one or two variables. However, the conclusions are easily adaptable to functions of n variables.

1. Figure 9 shows a very general function. It is obvious that there are several maxima and minima and since we seek the absolute mihima, there will have to be some testing done because calculus methods cannot seek other than relative extremes.

2. The method described above assumes an unbounded function. Therefore, from Figure 9, it is obvious that after all this effort of extreme seeking technique, it might turn out that none of the locations found is the absolute minimum.

3. The method assumes continuous functions in order to obtain derivatives. As seen on Figure 10, it is conceivable that the hypre-surface (surface in n dimensional space), made of several well-behaved functions on their own, will still have discontinuous inter-sections.

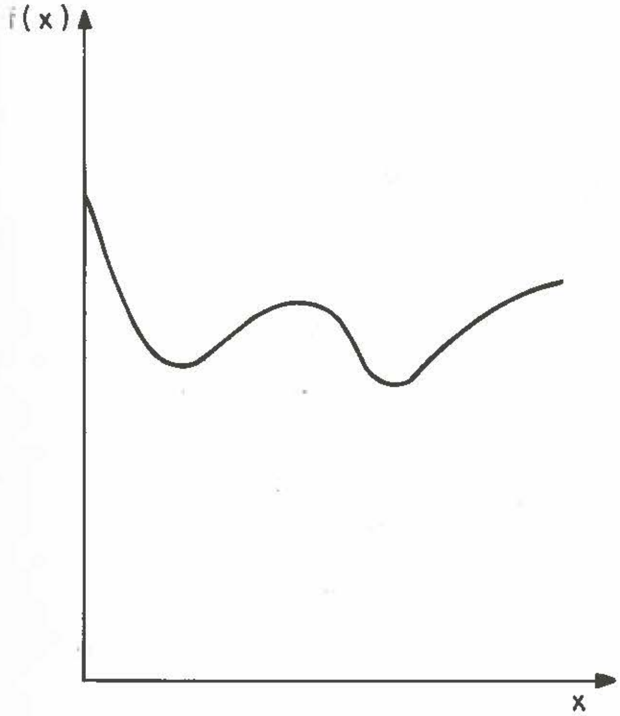

Fig. 9 A General One Dimensional Function

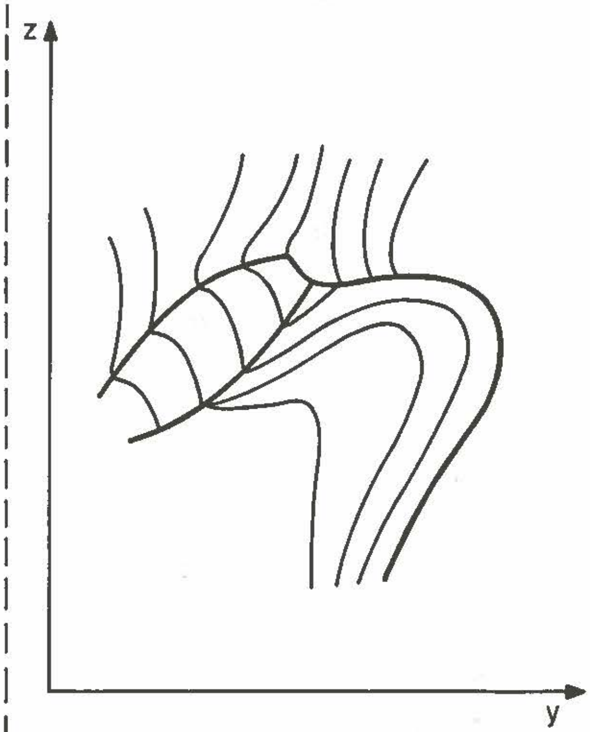

Fig. 10 Discontinuties arising from Intersections of Continuous Surfaces

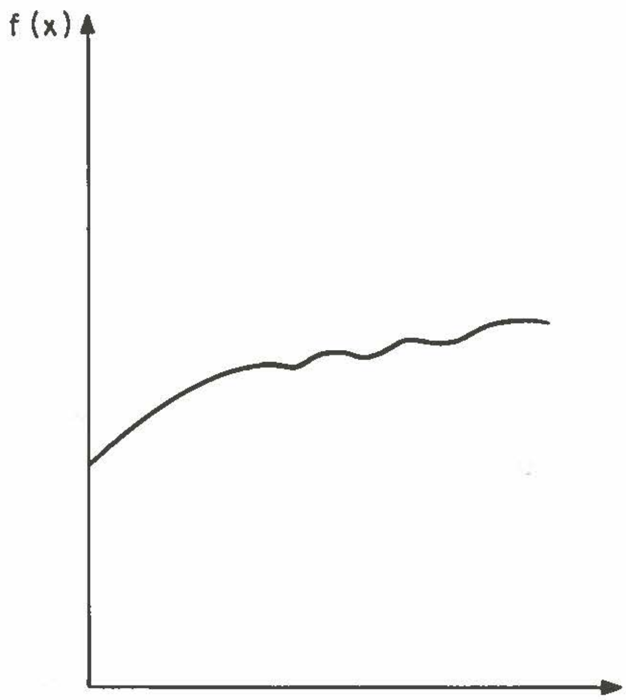

Fig. 11 Noise Type Functional Variation

4. Even though all functions and the hypresurface will be well behaved, the resulting set of simultaneous equations with the Lagrange multipliers will be nonlinear and probably coupled. This necessitates resort to complicated numerical methods for solution. Linearization is not applicable in this case since we are not considering a converging series of a small motion problem about an equilibrium position. Linearzing these equations would mean losing the whole significance of the problem.

5. Another consideration has to be adaptability of the method to a digital computer which, in this case, is impossible in the normal computer languages since they can manipulate only numbers. This means that if one wanted to computerize the closed form method, 90% of the solution would have to be done without the aid of a computer and the only task left for the computer would be routine numerical calculations essentially taking the place of a desk calculator; a wasteful use of a digital computer.

6. A recently developed language by IBM called "Formac" could overcome most of the difficulties mentioned in (5), since it enables symbolic solution of mathematical problems. At first, this language raised the hope of programming a general optimization problem where both function and constraints could have been input data in a functional form. However, the last step in the solution, i.e., the solution of the simultaneous nonlinear algebraic equations, eliminated this possibility also.

7. The last drawback of such a method would result from having to evaluate such points as shown in Figure 11. They are obviously "noise" type information which is entirely uninteresting as far as the design optimization problem is concerned.

(b) Steepest Ascent or Descent Optimization Technique

The steepest ascent or descent method as summarized by Gall, Ref. [2], uses the following logic:

1. Determine the partial derivatives with respect to each of the n dimensions at the present position which is arbitrary. This is usually done by calculating the value of the function whose minimum is sought at small increments on either side of the present position. The average partial derivatives through the present location can then be calculated for the particular dimension. This is repeated for all n dimensions.

2. Determine a new location by choosing increments for each of the n dimensions proportional to their own partial derivatives. (The directions chosen for each dimension, of course, depends upon whether a maximum or minimum is being sought.) This particular choice of increments forces the new location to be in the direction of the steepest path which passes through the old location.

3. Steps 1 and 2 are repeated until a local maxima (or minimum) is reached. In actuality, the search is stopped when all the partial derivatives are below some predetermined level, since otherwise the search could go on indefinitely looking for exactly zero partial derivatives.

4. Repeat steps 1 through 3 for several starting points. This is necessary since each starting point can only result in the determination of a single extremum. In most multidimensional problems, there will be several relative extrema.

This method has several drawbacks:

a. The partial derivatives have to exist; i.e., no discontinuities are allowed.

b. A high value of slope would indicate large increments; therefore, if the extreme is located at the intersection of two hypersurfaces, the method will oscillate back and forth. However, if the function is well-behaved and unimodal, this method is quite efficient.

c. The process is time-consuming since for each trial point 2^n additional points have to be evaluated where n is the number of dimensions the function depends on.

(c) Dynamic Programming

In any search for optimization techniques applicable to the problem at hand, it is imperative that dynamic programming be considered. Its phenomenal reduction of dimensionality when applicable, Ref. [4], the resulting freedom from the form of the expressions involved, and its almost automatic generation of all manner of conditional optima indicate that some effort be expended in evaluating this technique.

In order to apply dynamic programming, two conditions must be met:

1. One must be able to order the decision-making process in such a way that the state of the system after each decision can be described by a small number of parameters.

2. Any decision in the sequence depends only on the present state of the system and not on past states (Markovian property).

Of course, it is always possible to get the Markovian property by adding more parameters. This is useful only if the resulting system does not violate 1.

Can the ship design process be organized in such a way that conditions 1 and 2 are met? Strictly speaking, this question is vacuous. What we really want to know is: With our present state of knowledge of the interactions between the various subsystems that make up the ship design process and, most important, with our present state of knowledge of our own desires, can we so order the design problem? The answer to this question is probably not. To get a feel for this, let us examine two restricted formulations of the problem for which dynamic programming might be a considerable aid and see what is required.

1) Let us suppose that given the owner's requirements we somehow determine a feasible weight, space, and moment. Let us suppose further that we can measure the "return" of each system by the negative of the cost of that system which is a function of weight, space, and moment of that subsystem. Then, if our intention is to maximize this return, we have a fairly simple three-simensional dynamic program--the problem being to allocate the available weight, space, and moment among the various subsystems in such a manner as to minimize cost. The

owner's requirements will enter in the minimum that each of these quantities can assume for each subsystem. This formulation has two obvious drawbacks:

a) It does not tell us how to get the feasible solution which if arrived at in some intelligent manner will probably be fairly close to the optimum anyway.*

b) Our objective in ship design is rarely to simply allocate a certain amount of money among the various components, <u>but to minimize the total</u>.

2) Let us suppose we somehow determine a feasible combination of dollars, weight, space, and moment. Suppose further, that we can characterize each subsystem by an "efficiency" which is a function of the dollars, weight, space, and moment allocated to that subsystem. Suppose finally, that we can agree that the overall value of the design can be characterized by some function of these efficiencies, which function we shall wish to extremize. This can be set up as a four-dynamic program.** This formulation is also open to objection (a) above--how do we get feasible solution? It replaces objection (b) by its more basic form--can we agree on the subsystem and overall system "efficiencies" required. This last, of course, is not a dynamic programming problem; but until the answer is "yes", it is not quite clear how we can utilize this technique without complicating matters more than it is necessary.

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* Notice, if the solution offered is unfeasible, the dynamic program will discover it fairly quickly. It will also yield some information on how unfeasible it is. This opens up the possibility of an iterative procedure utilizing dynamic programming to obtain the feasible (and thus the optimum) solution.

** It does not mean that solving 3 and 4 dimensional dynamic programs is very straightforward. Full use of the Lagrange multiplier techniques is one way. On the other hand, by conventional methods we would have a 4 x N dimensional problem where N is the number of subsystems.

APPENDIX B

(Based largely on Ref. (1))

Empirical Equations, Design Constants and Design Factors

(A) Empirical Equations

These equations are the basis for all computer results and are included for illustrative purpose only. These equations could undoubtedly be presented in varying forms with varying degrees of accuracy. Unless stated to the contrary, all lengths and speed/length ratios are based on "Length-on-Waterline" (L.W.L.). In equations where "Length Between Perpendiculars" (L.B.P.) is required, L.W.L. is modified by a suitable Design Factor (K9).

Eq. #1 Used to determine optimum midship coefficient (C_x) for any given speed/length ratio.

$$CX = 0.977 + 0.018\ (V/\sqrt{L}) + 0.076\ (V/\sqrt{L})^2 - 0.115\ (V/\sqrt{L})^3$$

Eq. #2 Used to determine fuel oil capacity (tons). This is an exact relationship for the amount of fuel oil (WF) required to sail a given distance (R) plus 10%, at a given speed (V). The equation depends on a relationship of Fuel Rate vs. Shaft Horsepower - see equation no. 20 (F.R. = Fuel Rate)

$$WF = \frac{(1.10R) \times SHP \times F.R.}{2240 \times V} = 0.491 \times 10^{-3}\ \frac{R \times SHP \times F.R.}{V}$$

Eq. #3 Used to determine gross bale cubic of the ship. This curve applies to all dry cargo ships except where excessive sheer is used. This expression includes machinery space volume and excludes the double bottom and peak tanks.

$$G.B.C. = 0.875 \left[L \times B \times D \times C_B \right] \quad K9$$

Eq. #5. Used to approximate fuel oil capacity of the double bottom (WFB). This assumes LBP = K9 x LWL. This equation must be modified for excessive tank top heights and different fuel types.

$$WFB = \frac{1}{37.2} \left[(K9 \times L) \times B \times (K_6 \times D) \times (0.69\ C_B) \right]$$, where

C_B = Block Coeff. at L.W.L.; factor 60% is a correction for, (a) structure in inner bottom, and (b) correction to obtain C_B at the W.L. height equal to tank top height.

F.O. stowage factor = 372. Ft^3/L.T.

$$WFB = 18.55 \times 10^{-3} \left[L \times B \times D \times C_B \right] K_6 \times K9$$

$$= 21.20 \times 10^{-3} \left[\text{G.B.C. from Eq. 3} \right] K6$$

Eq. #6 Approximation to outfit weight,

$$W_o = 0.15 \ \frac{(L \times B)\ K9}{100}^{1.60}$$

Eq. #7 Approximation to steel weight,

$$W_s = 2.107 \ \frac{L\ (B - D) \times K9}{100}^{1.19}$$

Eq. #8 Approximation to "wet" machinery weight,

$$W_m = 7.18 \ \ SHP^{\ 0.495}$$

Eq. #9 Approximation to unit outfit cost; "Cost Point per Ton of

#10 Outfit vs. Tons of Outfit".

For: $\underline{0 \leq W_o < 1400:}$

$$P_{ot} = 1100 - 0.043\, W_o + (0.112 \times 10^{-3})\, W_o^{\,2}$$

$$- (0.1323 \times 10^{-6})\, W_o^{\,3}$$

For: $\underline{1400 \leq W_o < 2600}$

$$P_{ot} = 2430 - 1.928\, W_o$$

$$+ (0.722 \times 10^{-3})\, W_o^{\,2} - (0.091 \times 10^{6})\, W_o^{\,3}$$

For: $\underline{W_o \geq 2600}$

$$P_{ot} = \text{Constant}$$

$$= 698.8 \text{ cost points/L. ton outfit}$$

Eq. # 11 Approximation to steel cost. "Cost Points per Ton Steel vs. Tons Steel".

$$P_{ST} = 218.4 - 21.38 \left(\frac{W_s}{1000}\right) + 2.061 \left(\frac{W_s}{1000}\right)^2 -$$

$$- 0.1149 \left(\frac{W_s}{1000}\right)^3$$

Eq. #12 #13 Approximation to machinery cost. "Points per SHP vs. SHP" (SHP is normal shaft horsepower).

For: $\underline{SHP \leq 13.000:}$

$$P_{MT} = 137.7 - \frac{SHP}{75.32 + 5.92 \frac{(SHP)}{1000}}$$

For: $\underline{SHP > 13.00:}$

$$P_{MT} = \frac{SHP}{3.249\,(SHP) - 173.95}$$

Eq. # 14 Exact equation for the freeboard available, in inches. Equation assumes 3" margin line and that the uppermost continuous deck is Freeboard Deck, (i.e. Full Scantling Ship). Equation will differ for a shelter deck ship.

$$F_{BA} = 12 \left[D - (0.25 + H) \right]$$

Eq. #15 #16 #17 Equations for the curve of minimum permissible freeboard from U.S.C.G. Load Line Regulations. (Length L.W.L. is assumed equal to length on summer load line).

For: $\underline{L \leq 400:}$

$$F_A = 4.21 + 3.59 \frac{(L)}{100} + 3.71 \left(\frac{L}{100}\right)^2$$

For: $\underline{400 \quad L \quad 750:}$

$$F_A = - \; 77.67 + 42.58 \frac{L}{100} - 0.60 \left(\frac{L}{100}\right)^2$$

Eq. #18 Approximation to the inertia coefficient of the design load water plane.

$$\alpha = 0.0957 \times C_P - 0.0122$$

Eq. #19 Approximation to KG of fuel oil in deep tanks and F.O. Settlers. For normal ships this is taken as the tank top height + 60% of an assumed deck height.

$$KG_{FD} = K_6 D + 4.80$$

Eq. #20 Fuel oil rate:

Fuel Rate: $F.R. = \dfrac{0.5 \text{ SHP}}{\text{SHP} - 855}$

Eq. #21 Approximation to KG of fuel oil in double bottom. For normal ships this is taken as (2/3) x (tank top height).

$$KG_{FB} = 0.67\ K_6\ D$$

Eq. #22 Approximation to machinery KG, with boilers full and for conventional arrangement for steam turbine plant.

$$KG_M = 0.55\ D$$

(B) Design Constants

In the preparation of a program of this type, there are many design constants used which do not vary to any great extent for the particular type of ship under consideration. Therefore, it seems advisable to list these constants here.

a. Reynolds No. for S.W., 59° F:

   $Re = 1.3177 \times 10^5\ (L \times V)$

b. Roughness allowance: 0.0004

c. Fuel oil settler capacity = 150 L. tons.

d. Mass density of salt water @ 59° F. = 1.9905

e. F.O. density = 37.2 cu. ft./L. tons.

f. Basis for cost calculations are straight line depreciation plus average interest, 25 year economic lift, 2 1/2% scrap value, 5% simple interest.

g. Wetted Surface Factors with corresponding Beam/Draft Ratios:

W.S. Factor $= \sqrt{35}$ x W.S. Coeff.

B/H	Wetted Surface Factor
2.25	15.086
2.75	15.046
3.25	15.115
3.75	15.293

(C) Design Factors

The factors are defined as follows:

K1 = Correction to bare hull EHP for appendage resistance, propulsive coefficient and service margin. The appendage resistance and P.C. will be a function of ship type and number of propellers. An additional correction for percentage (%) over or under the equivalent Taylor Standard Series model could be used if the comparative merits of the parent form are known. Typical values for the above allowances are:

(a) Appendage allowance = 3% x bare hull EHP

(b) Propulsive coefficient = 75%

(c) Service margin = 25%; i.e., 1.25 x SHP = Service SHP

Using these allowances:

Normal SHP = (K1) x EHP (bare hull)

$$= \frac{(1.25)\ (1.03)}{(0.75)} \times \text{EHP} = 1.72 \times \text{EHP}$$

where K1 = 1.72

K2 = Approximation to the Vertical Center of Bouyancy. For normal forms it is taken as 54% of the draft (.54H). (K2 = 0.54)

K3 = Approximation to the Vertical Center of Gravity for steel weight; for normal ships this is taken as 61% of the depth to main deck (.61D). (K3 = 0.61)

K4 = Approximation to Vertical Center of Gravity for outfit weight. For normal ships this is taken as equal to the depth (D). (K4 = 1.00)

K5 = Cost points for fuel oil; units are: Points per one long ton.

K6 = Approximation to Tank Top height. For normal cargo ships this is taken as 1/9 depth. Therefore, tank top height = K6 x D, where K6 = 0.11.

K7 = Approximation to the Vertical Center of Gravity of cargo weight. Homogeneous loading is assumed. For normal ships this is taken as 63% of the depth (.63D). (K7 = 0.63).

K8 = Approximation to Vertical Center of Gravity of miscellaneous deadweight items. For normal ships this is taken as equal to the depth (D). (K8 = 1.00).

K9 = Ratio for modifying load waterline length to length between perpendiculars; L.B.P. = K9 x L.W.L.

K10 = "SHIP USE FACTOR" for the determination of fuel oil cost points for one year's operation.

The factor (K10) is derived as follows; let

V = Required speed (Owner's Requirement).

R = Range between fueling ports. (Owner's Requirement).

Wf = Tons fuel oil needed for steaming a distance 1.10R; this includes a 10% margin for reserve, see equation number 2; tons fuel oil required to steam a distance R at speed V (one trip) = 0.91 Wf.

Uf = Percentage of time (year) ship is operating at full power.

Ur = Percentage of time (year) ship is operating at reduced power.

r = Ratio of "fuel consumption at reduced power" to "fuel comsumption at full power", $(0 < r < 1)$.

Tons fuel oil required for one hour's operation at full power

$$= \frac{0.91 \times Wf \times V}{R}$$

Hours per year at full power = (Uf) (365) (24) = (Uf) (8760)

Hours per year at reduced power = (Ur) (8760)

Tons of fuel required for one hours operation at reduced power =

$$(.91\ Wf)\ \frac{(V)}{R}\ (r).$$

Therefore, the total tons fuel oil required per year:

$$(W_{fy}) = (U_f)\ (8760)\ (.91\ W_f)\ \frac{(V)}{R} + (U_r)\ (8760)\ (.91\ W_f)\ \frac{(V)}{R}\ (r) =$$

$$7971.6\ (U_f + U_r r)\ \frac{W_f V}{R} = K11\ \frac{W_f V}{R} = W_{fy}$$

where K10 = $7971.6\ (U_f + U_r r)$ = "Ship Use Factor" and Pf = total cost points for fuel oil required for one year's operation

$= W_{fy} \times K_5$ $\qquad P_f = \frac{K_{10} \times W_f \times v \times K_5}{R}$

Example:

For a conventional dry cargo ship, it is determined from a study of its proposed service, speed, and cargo handling capability that it will be at sea 55% of the time at a given fuel rate (see equation no. 20), be operating in port at minimum power (1/24 normal fuel rate) 35% of the time, and will have a "dead plant 10% of the time". On this basis,

U_f = 0.55, Ur = 0.35

$r = \frac{1}{24} = 0.042$, K10 = 4500

Cost of reserve fuel oil, which is encountered only once (during first refueling), is considered negligible when amortized throughout life of ship.

K11 = GM/Beam Ratio, minimum acceptable limit. For conventional dry cargo ships this will be taken equal to 0.05. For special type ships, this value may be varied to evaluate its influence on the "Optimum" ship characteristics. It should be noted that the GM derived from this ratio will be for the ship in the "Full Load Departure Condition", and uncorrected for free surface.